

## 5 Imperceptible Intersections: The Chopstick Illusion

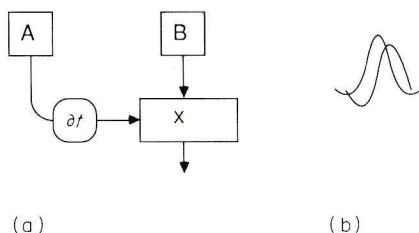
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How do we know in which direction a visual object is moving? A single receptor is not enough, since it registers only flicker without direction when a contour moves across it. So to sense direction you need two nearby receptors coupled together into a directionally selective unit. But directional selectivity alone cannot reveal the direction in which an object moves. A single local reading of velocity is not sufficient to recover the true motion in an image, because a moving line viewed through a moving aperture could be moving in any of a wide ( $180^\circ$ ) range of directions. We shall give a brief review of models of directional selectivity and of solutions to the so-called "aperture problem" and then turn to a new phenomenon which we call the "chopstick illusion". The visual system is unable to sense the motion path of the *intersection* of two moving lines.

### DIRECTIONAL SELECTIVITY

Reichardt (1961) proposed a model of directional selectivity based upon his ingenious experiments on motion detection in the insect eye. The output from a receptor A (Figure 1) is delayed and then compared with the undelayed output from a nearby receptor B. When the transit time for the contour to move from A to B is equal to the internal delay  $dt$  then the output from the comparator is maximum and the whole unit responds. The unit is silent when a contour moves in the null direction from B to A. Reichardt's comparator was a multiplicative correlator. Barlow & Levick (1965) suggested that the rabbit retina makes comparisons based on subtractive inhibition, and this gives similar results. More recent models of

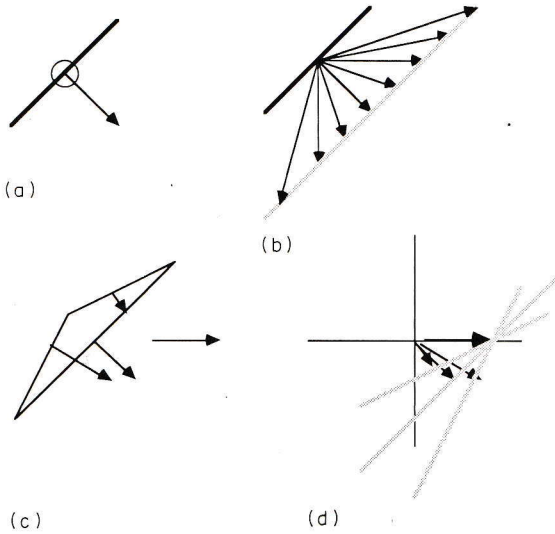


**Figure 1.** Generic direction-selective motion sensor. (a) When a contour passes from A to B (the preferred direction) the output of A is delayed then compared with the output of B. When the two outputs match the unit responds. Comparison could be a multiplicative correlation (Reichardt, 1961) or subtractive inhibition (Barlow & Levick, 1965). (b) Possible receptive fields for receptors A and B.

directional selectivity examine motion separately in different spatial-frequency bands, as proposed by Marr & Ullman (1981), van Santen & Sperling (1984), Adelson & Bergen and Watson & Ahumada (1985). Some of the evidence is discussed by Moulden & Begg (1986) and by Anstis (1988), and van Santen & Sperling (1985) have written a good critical review.

## THE APERTURE PROBLEM

As Adelson & Movshon (1982) have pointed out, a single local reading of velocity is not sufficient to recover the true motion in an image; to disambiguate the so-called aperture problem, readings from at least two moving straight lines are needed. When a moving line is viewed through an aperture (Figure 2a), or through the receptive field of a directionally selective unit, only the motion orthogonal to the line is visible because motion parallel to the line causes no change in the stimulus. Because there is a family of physical motions of various directions and speeds that appear identical, the motion of the line is ambiguous. Any of the physical motions indicated by the arrows in Figure 2(b) will appear the same when viewed through the aperture. The situation can be depicted graphically in "velocity space", a space in which each vector represents a velocity: the length of a vector corresponds to speed, and its angle corresponds to direction. The motion of the line is consistent with a family of motions that lie along a line in velocity space; this line is parallel to the moving line and orthogonal to the vector representing its "primary" motion. This effect was originally called the barber-pole illusion (Wallach, 1935) but is now more generally known as the aperture problem (Ullman, 1979). It applies equally well to an observer viewing a line through a hole in a shutter, as to a directionally selective motion-sensitive neuron (reviewed by Berkley, 1982) which views moving contours within the small region bounded by its receptive field. One might think that it would be easy, given such neurons, to compute the velocity at each region in the visual field, but this is not so. If two moving gratings are superimposed (Adelson & Movshon, 1982), the resulting tartan or plaid pattern moves with a speed and direction that can be predicted from

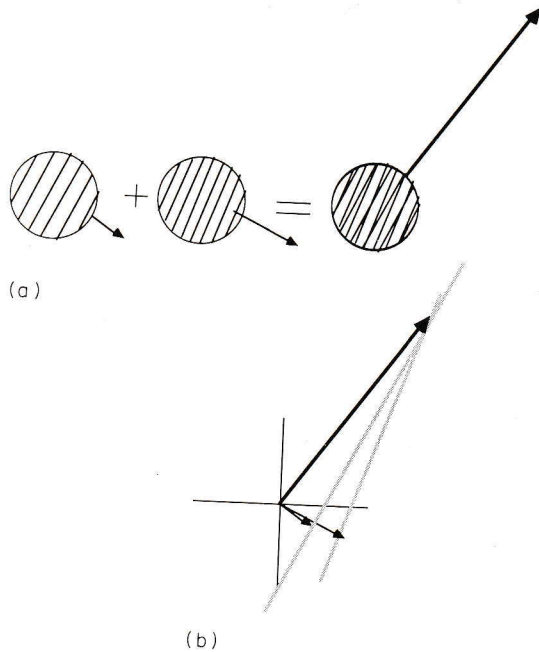


**Figure 2.** (a) A moving line viewed through a stationary aperture is ambiguous. (b) Its true motion could be as shown by any of the arrows (vectors). The length of each arrow represents speed. Note that all arrowheads lie on the grey line—the “velocity constraint line”. (c) Motion of each of three sides of a triangle is ambiguous, as in (a). (d) However, the velocity constraint lines all meet at one point which defines the true motion of the triangle (horizontal arrow) (after Adelson & Movshon, 1982; Movshon *et al.*, 1985). Note that this direction is not the vector sum or average of the motion of the three lines.

the velocity space construction; the two loci of possible motions intersect at a single point, corresponding to the motion of the coherent pattern (Figure 3). Note that this velocity space combination rule is different from a vector sum or vector average (Figure 2c, d; Figure 3). In Figure 3, two gratings move down and to the right. Their vector sum or average also moves down and to the right, but the velocity space construction is a motion up and to the right, the motion that observers report when the gratings are seen to cohere. Nothing requires that coherent motion be seen, and instead of a single rigidly moving plaid, observers sometimes reported two transparent gratings sliding over each other in incoherent motion. In general, observers were more likely to report incoherent motion when the angle between the two gratings was made larger, as their speeds increased, and as the spatial frequency increased, although this spatial frequency effect was rather weaker than the others. Under ideal conditions (identical spatial frequencies, low speeds and a modest angle), the two gratings always cohered into a single moving plaid.

Movshon *et al.* (1983) proposed a two-stage model of motion perception. The first stage contained direction- and orientation-selective units with elongated receptive fields, which responded to motion of the component gratings. The second stage contained motion analysers, which combined the signals from several units in the first stage, sensing the intersection of velocity constraints by means of coincidence detectors or “and” gates. These analysers responded to the motion of the plaid. Movshon *et al.* found that 40% of neurons in area MT of the macaque





**Figure 3.** (a) Two gratings move down to the right at different velocities. The perceived velocity of the compound grating is not a vector sum, but is up and to the right. (b) Velocity constraint lines of the two moving gratings define the motion of the compound grating, up and to the right (after Adelson & Movshon, 1982).

were component direction selective and 25% were plaid direction selective. They obtained psychophysical evidence for their model with a  $2 \times 2$  adaptation experiment. Adaptation to a grating makes it harder to see a test grating of the same spatial frequency and orientation, so they attempted to adapt the first and second stages of their model differentially. Adaptation to component motion ought to elevate detection thresholds for the component gratings, whereas adaptation to pattern motion ought to affect coherence. Their adapting and test patterns consisted of a horizontally moving grating of vertical bars, and a horizontally moving diagonal plaid pattern composed of two oblique gratings. They found, for instance, that adapting to the vertical grating did not elevate the threshold for the component gratings but did reduce the perceived coherence of the pattern motion.

Do we really need a two-stage model of motion perception? Perhaps the visual system merely tracks the intersections of the two crossed gratings, that is, the moiré fringes. These fringes are beat frequencies that contain no Fourier energy, but if there were an early non-linearity in the contrast response function this could produce intermodulation distortions in the mixture of two sinusoidal gratings which would supply Fourier energy at the moiré fringe frequency and orientation. This would suffice to provide a motion perception using only very-low-level receptive fields. If this were the case, there might be no need to argue for the

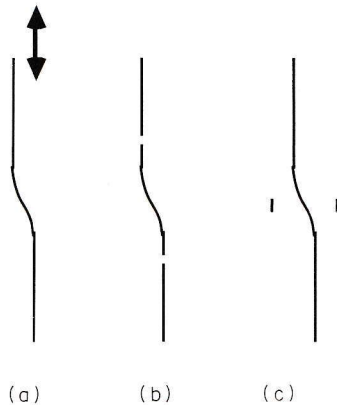


synthesis of differently oriented velocity signals. Furthermore, the moiré fringes move in just the direction predicted by the velocity constraint model, and the adaptation experiment described above does not disprove the moiré fringe hypothesis. However, Welch (1988) obtained experimental evidence against a moiré fringe story. She noted that speed discrimination can be much worse at low than at medium speeds. She measured just noticeable differences in velocity for plaids moving at speeds five times the component gratings, and found that speed discrimination followed the speed of the component gratings, not that of the moiré fringes. Speed discrimination for the plaids was not matched by a grating mimicking the speed and spatial separation of the fringes. These results support the velocity constraint model.

### Terminators Affect the Perceived Direction of Movement

Wallach (1935) viewed a moving grating through an elongated aperture. The motion appeared to run parallel to the long axis of the aperture. Shimojo *et al.* (1988) added stereoscopic depth, and found that motion parallel to the long edges was stronger if the grating lay in depth in front of the aperture because the grating appeared to be the same size as the aperture and the ends of the bars were perceived as "object terminators". Motion parallel to the long edges was weaker if the grating lay in depth behind the aperture because the grating appeared to extend behind the aperture, and the ends of the bars were perceived as "occlusion intersections".

Nakayama & Silverman (1988a, b) moved a vertically oriented cumulative Gaussian waveform (Figure 4) upward. This figure looked highly non-rigid when it moved. Moving "terminators", defined as the ends of line segments, dramatically increases the rigidity of the figure, especially if the segments are gaps "on" the line as in (b) rather than short line segments "off" the line, appearing as



**Figure 4.** (a) When this cumulative Gaussian curve is translated up and down in the plane of the page (arrow), it appears highly non-rigid. (b) (c) Adding terminators (ends of line segments) dramatically increases the rigidity of the figure, especially if the segments are gaps "on" the line as in (b) rather than short line segments "off" the line as in (c) (after Nakayama & Silverman, 1988b).

short line segments having the same length as the gaps. The figure now looked far more rigid, especially when the terminators lay “on” the line. The authors derived strong support from their results for Hildreth’s (1984) model of motion perception, in which local differences in velocity are integrated (smoothed) but only along contours. Hildreth’s scheme computes the velocity field which minimizes differences in velocity along contours yet is consistent with constraints dictated by the aperture problem. For our purposes, the powerful influence of terminators upon motion perception should be carefully noted.

In the aperture problem the motion of a stimulus is inherently ambiguous. We shall now describe some new illusions from moving stimuli: first an “Etch-a-Sketch phenomenon”, in which the stimulus motion of an intersection is unambiguous but the visual system is unable to sense the motion, and then a new “chopstick illusion”, in which motion of an intersection is misperceived because it is perceptually captured by motion of the terminators.

## THE ETCH-A-SKETCH PHENOMENON

It is easy to perceive the motion path of a small moving object. When a fly buzzes around in circles we can see the fly and we can also perceive the circles; and without much trouble we can read words written in the air by a moving finger, or written on a screen by a small cross-shaped cursor that leaves no visible trail behind it. Such small moving objects are unambiguous. But suppose that a word is being handwritten on a screen, not by a moving finger or cursor but by Etch-a-Sketch cross-hairs with long horizontal and vertical lines that extend off the edge of the screen, hiding the ends of the lines. As one watches the movement the two lines are not seen as a rigid cross but appear to slide over each other. Surprisingly enough, it is now almost impossible to read the word. Why? The centre of the cross traces out the same path as before, and all that is missing is the information from the terminators—the tips of the cross. Hiding them makes the word hard to read, yet when they were present they added no new information at all because they simply duplicated the movements of the centre of the cross. It seems that we pay more attention to the tips of lines (terminators) than we do to the crossings of lines (intersections). This is the basis of the chopstick illusion.

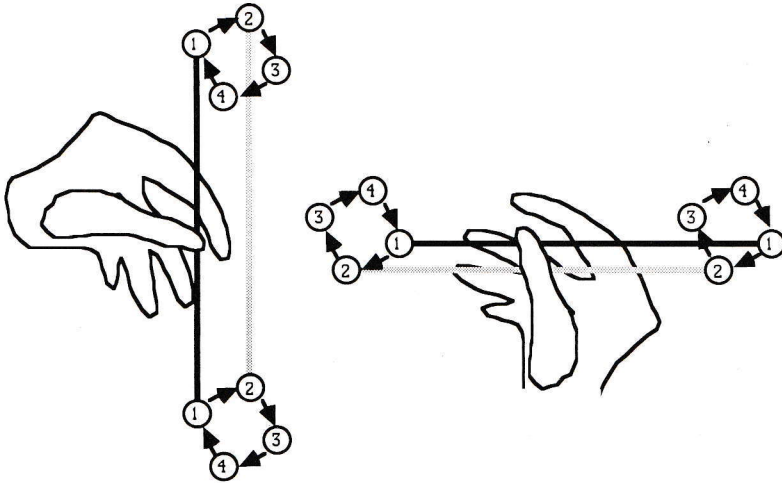
## THE CHOPSTICK ILLUSION

When a single line moves behind an aperture, the stimulus is ambiguous because there is not enough visual information to determine the true motion path. When a pair of crossed lines move orthogonally in the Etch-a-Sketch phenomenon, the path of the intersection is fully determined but we seem to be unable to perceive this unambiguous information. We shall now describe a “chopstick illusion” in which misleading information from the tips of two moving lines or chopsticks alters the perceived path of their intersection.

We put the tips of the lines (the terminators) into competition with their centres. The left hand holds a chopstick and moves it along a clockwise path while keeping it vertical, like a cabin on a Ferris wheel. The right hand moves a second chopstick along a clockwise path while keeping it horizontal (Figure 5). The two clockwise motions are in the same plane but in counterphase so that when the vertical chopstick moves through the highest point of its circular path the horizontal chopstick moves through its lowest point. When the chopsticks do not overlap, each one is seen veridically. However, when they overlap to form a cross so that they slide over each other, an illusion appears. The intersection of the two sticks physically traces out an anticlockwise path, like a Lissajous figure, but perceptually it appears to rotate clockwise, captured by the clockwise motion of the line tips (Figure 6). Thus the perceived motion of the tips propagates back to the intersections, which are incorrectly seen as moving along the same paths as the tips.

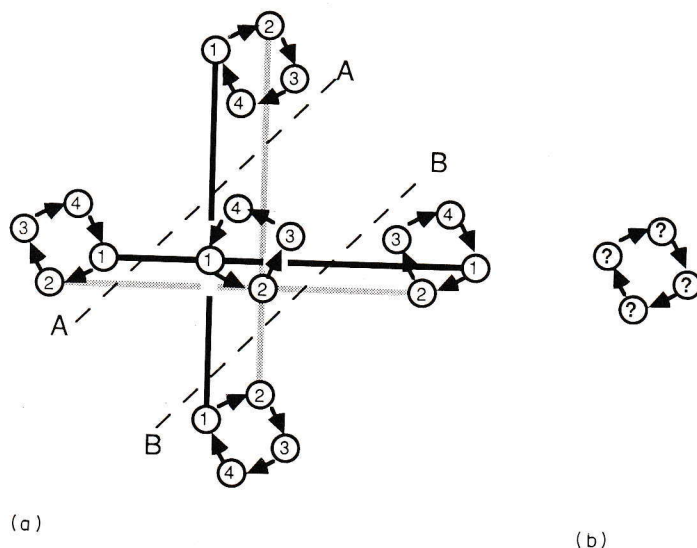
The static diagrams shown as figures are no substitute for seeing the motion, and the reader is encouraged to try out the effects by rubbing two pencils together. The skill can be picked up in a few moments. The illusion works just as well when the observer is moving the pencils while viewing them, and in fact the clockwise illusion is so compelling that it is often hard to accept that the intersections are really moving anticlockwise.

To give better stimulus control the chopsticks were replaced by a vertical and a horizontal line on a monitor screen controlled by a Commodore Amiga computer (Anstis, 1986). As before, each line moved along clockwise paths in counterphase, and the intersection, which actually traced out an anticlockwise path, appeared to rotate clockwise. This illusion was very robust; it was impossible to perceive the



**Figure 5.** To generate the chopstick illusion, the left hand holds a pencil or chopstick vertical and moves it along a circular clockwise path as shown by the numbers, keeping it always vertical. The right hand moves a horizontal chopstick along a clockwise path which is  $180^\circ$  out of phase with the vertical chopstick—compare the numbers in the small circles, which show the positions of the ends of the two chopsticks at times 1–4. Chopsticks are drawn in black for time 1 and in grey for time 2.

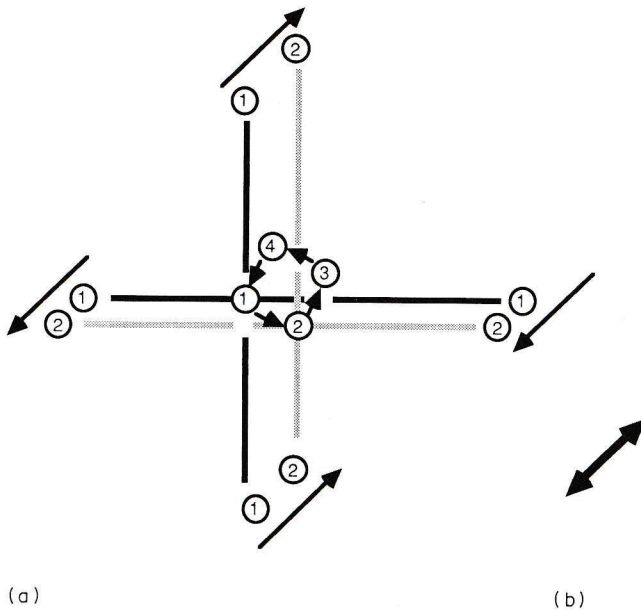




**Figure 6.** (a) Results of the chopstick illusion. The two moving chopsticks shown in Figure 5 now overlap at their centres. The motion path of the intersection is actually anticlockwise, as shown by the numbers at the centre. (b) However, the intersection gives the illusion of moving clockwise, under the influence of the paths of the terminators. Numbers in the circles are replaced by question marks because the order in which the intersection appears to pass through any points is not clear. Of course it does not actually pass clockwise through any points. The importance of the terminators is confirmed by clipping the ends of the chopsticks along the oblique lines A--A, B--B. Results of this are shown in Figure 7.

true anticlockwise rotation of the intersection. Also, the intersection did not look like a rigid cross (like two rods welded together), but the lines appeared to slide over each other. If the intersection was viewed through a small stationary circular window that hid the terminators, the illusion disappeared and one correctly perceived a rigid cross moving anticlockwise. However, the illusion reappeared at full strength as soon as the screen was removed.

Objectively, whenever the vertical line moved to the right then the intersection also moved right. When the horizontal line moved to the right it merely slid along its own length and the intersection stayed where it was. However, the subjective situation was the opposite of this. When the (tips of) the horizontal line moved to the right this percept propagated along the line and the intersection falsely appeared to move to the right. When the vertical line moved to the right the intersection was barely seen to move. In other words the horizontal component of the intersection's motion was objectively constrained by the horizontal oscillations of the vertical line, but it was subjectively constrained by the moving tips of the horizontal line. To verify this the ends of the lines in Figure 6 were clipped along the oblique lines A--A and B--B, giving the stimulus shown in Figure 7. This kept the central part of the cross untouched, as if viewed through an invisible window with oblique edges. Thus the tips of the vertical line moved obliquely up to the right while the tips of the horizontal line moved down to the left. As a result, the



**Figure 7.** (a) Stimulus is the same as in Figure 6 except that the terminators have been clipped along  $45^\circ$  oblique lines. The terminators of the vertical and horizontal chopsticks now move obliquely in antiphase. The central intersection moves along the same circular anticlockwise path as in Figure 5. (b) But now the intersection gives the illusion of moving back and forth obliquely.

intersection now also appeared to move obliquely, up-right and down-left, parallel to the terminators, although it was hard to perceive the phase of its motion. This illusion was moderately robust, and could be overcome by an effort of will. Viewing the intersection through a small stationary round aperture confirmed, as before, that it was still really moving anticlockwise.

### Rigidity and Coherence

When a rigid cross translates in its own plane, the distance from each terminator to the intersection remains constant. When two lines slide over each other these distances are continually changing. What kind of "rigidity assumptions", if any, does the visual system make?

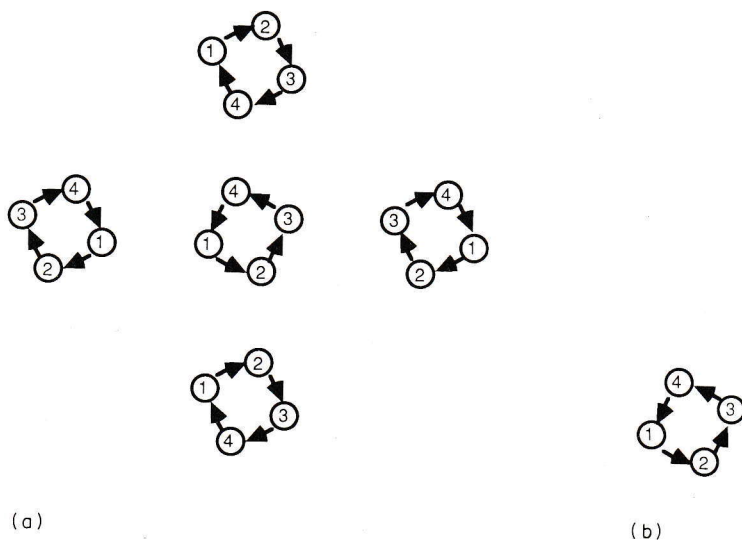
There are two possibilities; it might perceive either the intersection or the rods as rigid. In the first case it would perceive a rigid (glued or welded) intersection of two rubber rods as the rods changed in length. In fact this is never reported. Instead the visual system parses each line as being rigid (and unchanging in length, as the terminators assert), but the intersection is parsed as a non-rigid sliding region. We might say that when given a choice the visual system prefers to perceive sliding or occluding objects rather than rubbery objects. This may reflect the statistical properties of the physical world, in which objects constantly occlude and move past each other, but rubbery objects are uncommon.

*Necessary conditions*

The illusion occurred when the lines intersected at any angle between about  $30^\circ$  and  $90^\circ$ , and the lines could be solid or dashed, straight or curved. Even intersecting circles gave the illusion (not shown). Our standard intersections were X-junctions, but we found that T-junctions and L-corners were also captured by their terminators, although to a lesser extent. Link & Zucker (1988) discuss psychophysical sensitivity to static L-corners.

It is not the case that the outermost parts of the display simply capture the central parts, whatever these might be. If the tips of the lines and the intersection were each replaced by isolated spots (Figure 8) the illusion broke down and the motion of the central dot was seen veridically, and now it was seen not as an intersection but as a separate moving object, circling anticlockwise in the opposite direction to the terminator dots. Intersections are captured not merely because they happen to be in the central region of the stimulus, but because they are intersections.

The chopstick illusion does not result simply from the fact that terminators outnumber intersections. An array of eight parallel horizontal lines intersected with an array of eight parallel vertical lines. As before, the vertical and horizontal lines all moved clockwise but with a phase shift between them. This array had 64 intersections and only 32 terminators, but when the array was set in motion the terminators moving clockwise around the edges of the array still captured the intersections. As before, the intersections were really moving anticlockwise but appeared to move clockwise.



**Figure 8.** (a) Stimulus is the same as in Figure 6 except that the chopsticks have been removed. Terminators and intersection have been replaced by spots which circled respectively clockwise and anticlockwise. (b) Result: no illusion. The central spot was correctly seen as circling anticlockwise.



Terminators need not lie around the outer perimeter of the stimulus. "Blobs" and gaps in the interior of the figure are equally effective as terminators. The array of eight vertical and eight horizontal lines was made to fill the whole screen, which now acted like a large rectangular aperture hiding the ends of the lines. Now that the terminators were hidden by the edges of the screen, observers reported seeing a rigid square mesh circling coherently in an anticlockwise direction. This veridical percept showed that the chopstick illusion was absent. Next, gaps were introduced into the vertical and horizontal lines at four points surrounding one intersection. The gaps in the horizontal lines moved clockwise, and so did the gaps in the vertical lines but with the usual phase shift. Result: the single intersection that was surrounded by gaps appeared to move clockwise although all the other intersections still appeared to move anticlockwise. This shows that gaps in lines acted as efficient terminators and constrained the motion of the intersection that they surrounded. However, their influence was local and did not extend to more distant intersections. If gaps were added to surround every intersection then all the intersections showed the chopstick illusion and appeared to move clockwise. A curious phenomenon of direction of gaze was noted: with these gaps, all the intersections appeared to move clockwise when the centre of the screen was viewed, but if the gaze was deflected by some  $10^\circ$  so that the screen was viewed in peripheral vision, the intersections appeared to reverse in direction and move anticlockwise, in the veridical direction. The reason was that peripheral acuity was too low to resolve the gaps, which therefore lost their effectiveness. Blurring the screen with tracing paper in central vision had the same effect as using peripheral vision.

## DISCUSSION

### Comparison with Earlier Studies

Our results are really complementary to those of Adelson & Movshon (1982) in that they examined the conditions that gave coherent motion of two crossed gratings when they combined into a plaid, whereas we examined the conditions that gave our chopstick illusion, which is an instance of incoherent motion of two crossed lines as they slid over each other. Our percept of a rigid cross corresponds to their coherent motion of a (rigid) plaid, and our percept of rods sliding over each other corresponds to their incoherent motion of two transparent gratings. They showed that coherent plaids always seemed to move in the same direction as the moiré fringes or intersections of the two gratings. (This is a rule of thumb, not a model for their results.) Motion was always coherent except when their two gratings differed markedly in orientation, contrast, speed or spatial frequency. Since they always used a fixed circular aperture, line terminators had little systematic effect on their results. In our studies motion was nearly always incoherent and terminators played a crucial role. We asked (as they did not) what was the perceived direction of the intersections during incoherent motion. Stated differently, when two gratings cohere the two stimulus motions reduce to a single perceived motion. Our question was: when two lines (or gratings) do not cohere,

such that two transparent motions are seen, can a *third* motion be seen, namely the motion of the intersections? We find that the answer is generally no.

### **The Role of Covering and Uncovering**

If a man walks along in front of a picket fence, his body successively covers up pickets ahead of him and uncovers pickets behind him. As he walks to the right, pickets disappear on the right and other pickets reappear on the left. These pickets generate spurious signals of apparent motion which the visual system suppresses or ignores (Ramachandran & Anstis, 1986). Also, the edges of his body define a moving shape which is of perceptual interest, but the intersections of his curved or oblique body contours with the vertical edges of the pickets define occlusion intersections that do not correspond to any physical object. These intersections generate spurious signals of vertical movement up and down the pickets, which the visual system ignores. So the visual system's seeming inability to process moving intersections may be not a design fault but a useful and adaptive rejection of spurious motion signals whose uncritical acceptance would hamper, not help, our interpretation of a moving visual world.

## **ACKNOWLEDGEMENT**

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